National Weather Service
National Centers for Environmental Prediction

The WRF NMM Core

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NMM Dynamic Solver

- Basic Principles
- Equations / Variables
- Model Integration
- Horizontal Grid
- Spatial Discretization
- Vertical Grid
- Boundary Conditions
- Dissipative Processes
- Namelist switches
- Summary

Basic Principles

- Use full compressible equations split into hydrostatic and nonhydrostatic contributions
  - Easy comparison of hydro and nonhydro solutions
  - Reduced computational effort at lower resolutions
- Apply modeling principles proven in previous NWP and regional climate applications
- Use methods that minimize the generation of small-scale noise
- Robust, computationally efficient

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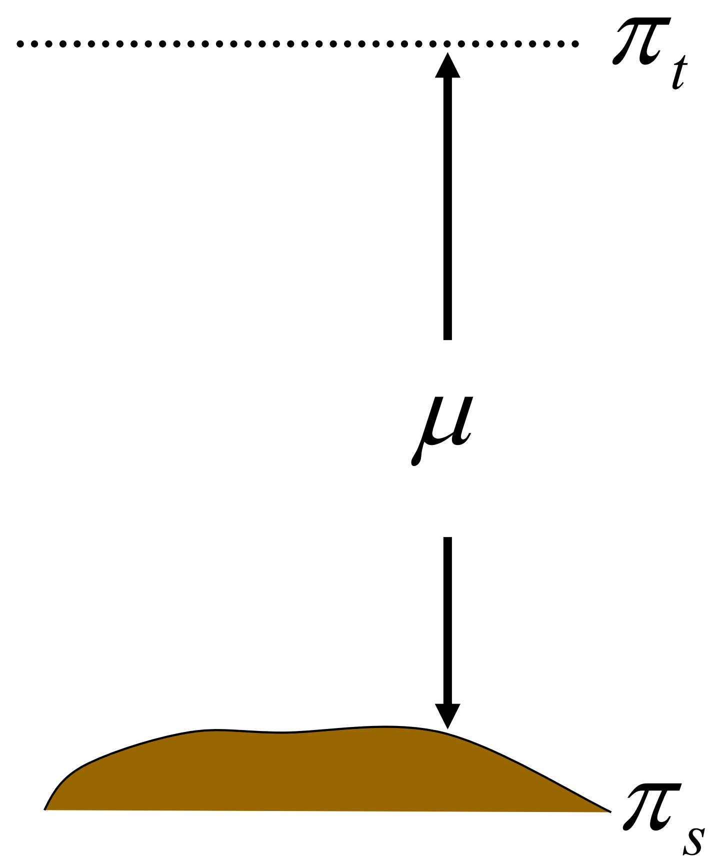
Mass Based Vertical Coordinate

To simplify discussion of the model equations, consider a sigma coordinate to represent a vertical coordinate based on hydrostatic pressure (π):



$$\mu = \pi_s - \pi_t$$

$$\sigma = \frac{\pi - \pi_t}{\mu}$$

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WRF-NMM dynamical equations
inviscid, adiabatic, sigma form

Analogous to a hydrostatic system, except for p and ε, where p is the total (nonhydrostatic) pressure and ε is defined below.

Momentum eqn.

$$\frac{\partial \mathbf{v}}{\partial t} = -\mathbf{v} \cdot \nabla_\sigma \mathbf{v} - \dot{\sigma} \frac{\partial \mathbf{v}}{\partial \sigma} - (1+\varepsilon) \nabla_\sigma \Phi - \alpha \nabla_\sigma p + f \mathbf{k} \times \mathbf{v}$$

Thermodynamic eqn.

$$\frac{\partial T}{\partial t} = -\mathbf{v} \cdot \nabla_\sigma T - \dot{\sigma} \frac{\partial T}{\partial \sigma} + \frac{\alpha}{c_p} [\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla_\sigma p + \dot{\sigma} \frac{\partial p}{\partial \sigma}]$$

Hydrostatic Continuity eqn.

$$\frac{\partial \mu}{\partial t} + \nabla_\sigma \cdot (\mu \mathbf{v}) + \frac{\partial (\mu \dot{\sigma})}{\partial \sigma} = 0$$

$$\boxed{\varepsilon \equiv \frac{1}{g} \frac{dw}{dt}}$$

$$\boxed{\alpha = RT/p}$$

Janjic et al. 2001, *MWR*

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Hypsometric eqn.

$$\frac{\partial \Phi}{\partial \sigma} = -\mu \frac{RT}{p}$$

Nonhydro var. definition (restated)

$$\varepsilon \equiv \frac{1}{g}\frac{dw}{dt}$$

3rd eqn of motion

$$\frac{\partial p}{\partial \pi} = 1 + \varepsilon$$

ε generally is small. Even a large vertical acceleration of 20 m/s in 1000 s produces ε of only ~0.002, and nonhydrostatic pressure deviations of ~200 Pa.

Nonhydrostatic continuity eqn.

$$w = \frac{1}{g}\frac{d\Phi}{dt} = \frac{1}{g}\left(\frac{\partial \Phi}{\partial t} + \mathbf{v} \bullet \nabla_\sigma \Phi + \dot{\sigma}\frac{\partial \Phi}{\partial \sigma}\right)$$

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Properties of system

- Φ, *w*, and ε are not independent → no independent prognostic equation for *w*!

- ε << 1 in meso- and large-scale atmospheric flows.

- Generically, the impact of nonhydrostatic dynamics becomes detectable at resolutions < 10 km, and important at ~1 km.

WRF-NMM predictive variables

- Mass variables:
  - `PD` – hydrostatic pressure depth (time/space varying component) (Pa)
  - `PINT` – nonhydrostatic pressure (Pa)
  - `T` – sensible temperature (K)
  - `Q` – specific humidity (kg/kg)
  - `CWM` – total cloud water condensate (kg/kg)
  - `Q2` – 2 * turbulent kinetic energy (m^2/s^2)
- Wind variables:
  - `U,V` – wind components (m/s)

Model Integration

- Explicit time differencing preferred where possible, as allows for better phase speeds and more transparent coding:
  - horizontal advection of u, v, T (2nd order Adams-Bashforth)
  - advection of q, cloud water, TKE ("passive substances")

- Implicit time differencing for very fast processes that would require a restrictively short time step for numerical stability:
  - vertical advection of u, v, T (Crank-Nicolson) and vertically propagating sound waves

Model Integration

Advection of passive substance (Q, Q2, CWM)

- Positive definite approach similar to Janjic (1997) scheme used in Eta model:
  - starts with a Lagrangian upstream advection step
  - anti-diffusion/anti-filtering step to restore steepness, optimized to reduce dispersiveness
  - conservation enforced after each anti-diffusion step (allows for open boundary conditions)
  - Typically called IDTAD times as many time steps as the rest of the dynamics, but *HWRF calls it each step*

Model Integration

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Fast adjustment processes - gravity wave propagation

Forward-Backward: Mass field computed from a forward time difference, while the velocity field comes from a backward time difference.

In a shallow water equation sense:

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x};\frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x}$$

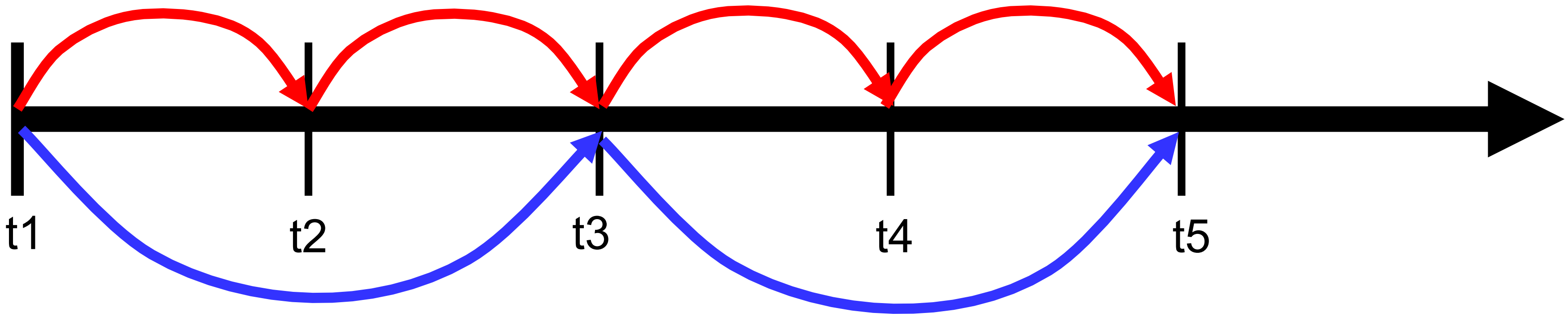
$$h^{\tau+1} = h^{\tau} - \Delta t\, H\frac{\partial u^{\tau}}{\partial x}$$

$$u^{\tau+1} = u^{\tau} - \Delta t\, g\frac{\partial h^{\tau+1}}{\partial x}$$

Mass field forcing to update wind from $\tau+1$ time

Ames, 1968; Janjic and Wiin-Nielsen, 1977; Janjic 1979

All dynamical processes every fundamental time step, except….



…passive substance advection, traditionally every other time step. HWRF each timestep, though.

Model time step "dt" specified in model namelist.input is for the fundamental time step.

Generally about 2.25x** the horizontal grid spacing (km), or 350x the namelist.input "dy" value (degrees lat).

** runs w/o parameterized convection may benefit from limiting the time step to about 1.9-2.0x the grid spacing. ***I believe HWRF runs closer to ~1.5x the grid spacing.***

Now we'll take a look at two items specific to the WRF-NMM horizontal grid:

- Rotated latitude-longitude map projection (only projection used with the WRF-NMM)
- The Arakawa E-grid stagger

Rotated Latitude-Longitude

- Rotates the earth’s latitude & longitude such that the intersection of the equator and prime meridian is at the center of the model domain.

- This rotation:
  - minimizes the convergence of meridians.
  - maintains more uniform earth-relative grid spacing than exists for a regular lat-lon grid.

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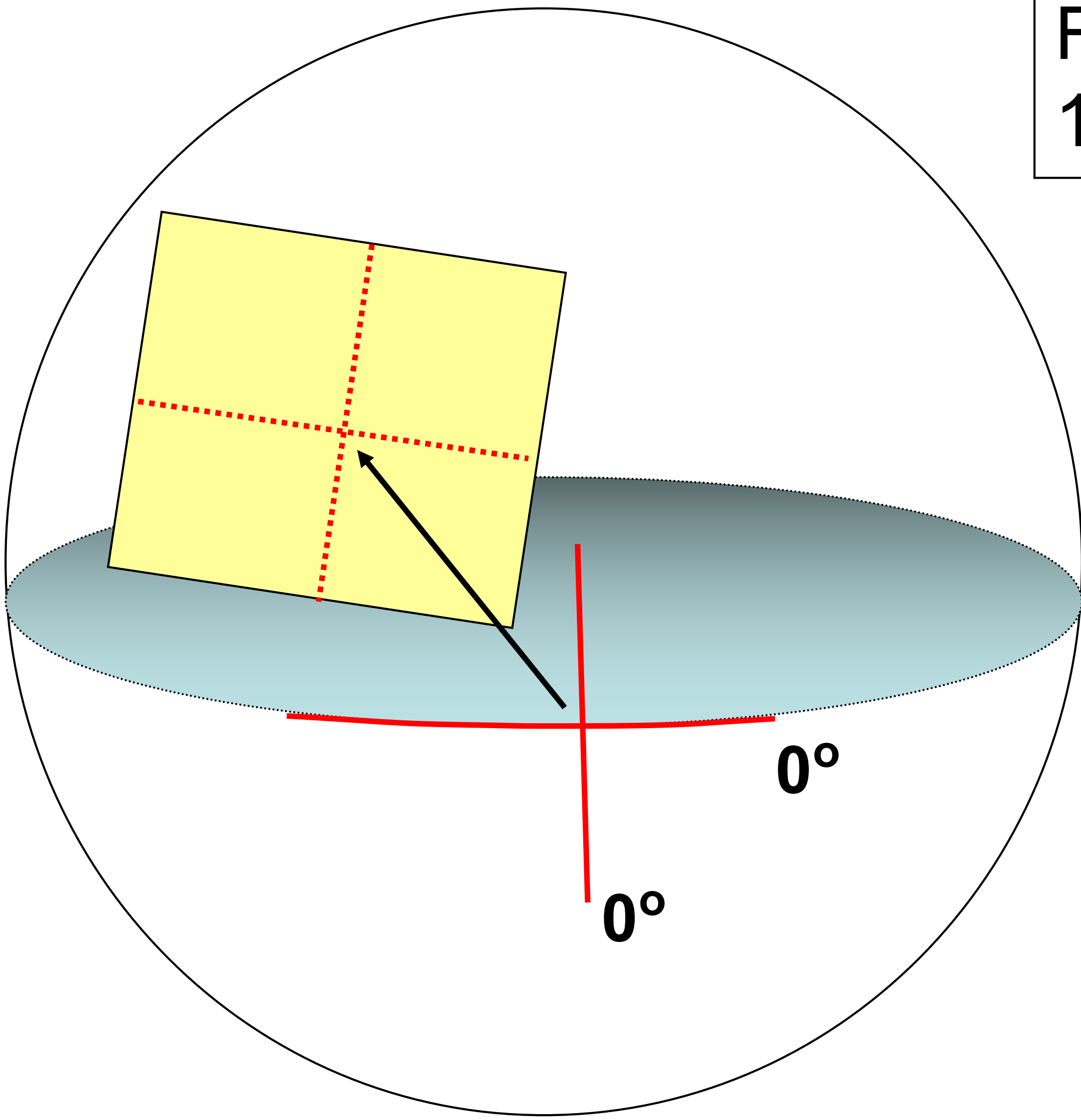
Impact on variation of Δx over domain



For a domain spanning 10N to 70N:

$$\Delta x \propto \cos(lat)$$

Regular lat-lon grid

$$\cos(70^0)/\cos(10^0) = 0.347$$

Rotated lat-lon grid

$$\cos(30^0)/\cos(0^0) = 0.866$$

The E-grid Stagger

H	v	H	v	H	v	H	(v)
v	H	v	H	v	H	v	(H)
H	v	H	v	H	v	H	(v)
v	H	v	H	v	H	v	(H)
H	v	H	v	H	v	H	(v)

H=mass pt, v=wind pt

red=(1,1) ; blue=(1,2)

XDIM=4 (# of mass points on odd numbered row)

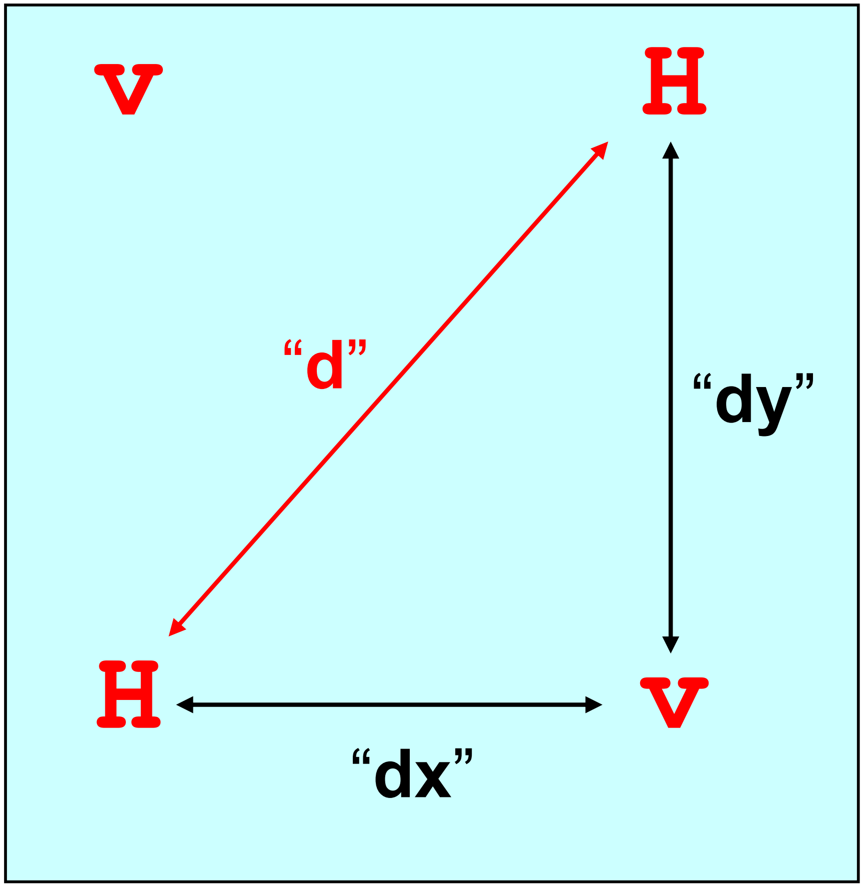
YDIM=5 (number of rows)

The E-grid Stagger - properties

- Due to the indexing convention, the X-dimension is half as large as would be expected from a C-grid domain (typically XDIM < YDIM for the E-grid).
- "Think diagonally" –the shortest distance between adjacent like points is along the diagonals of the grid.

- E-grid energy and enstrophy conserving momentum advection scheme (Janjic, 1984, MWR) controls the spurious nonlinear energy cascade (accumulation of small scale computational noise due to nonlinearity) more effectively than schemes on the C grid – an argument in favor of the E grid.

The E-grid Stagger



- Conventional grid spacing is the diagonal distance "**d**".

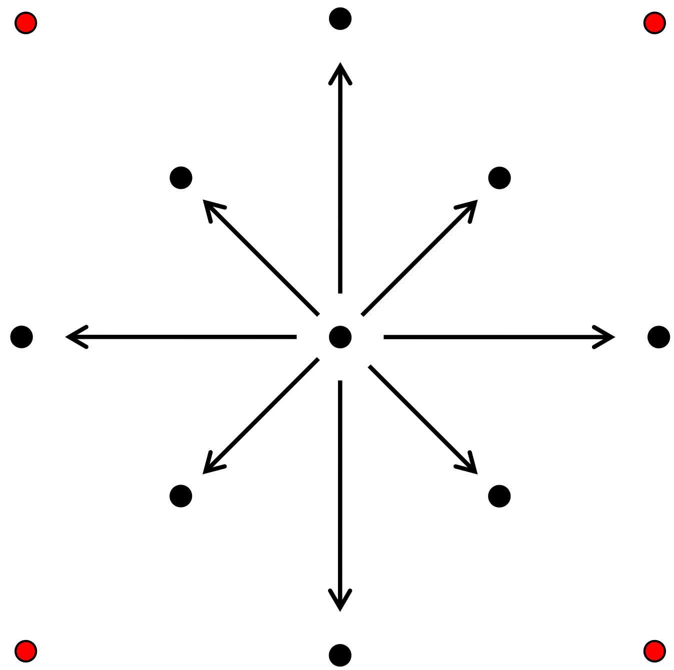
- Grid spacings in the WPS and WRF namelists are the "**dx**" and "**dy**" values, *specified in fractions of a degree for the WRF-NMM*.

Spatial Discretization

General Philosophy

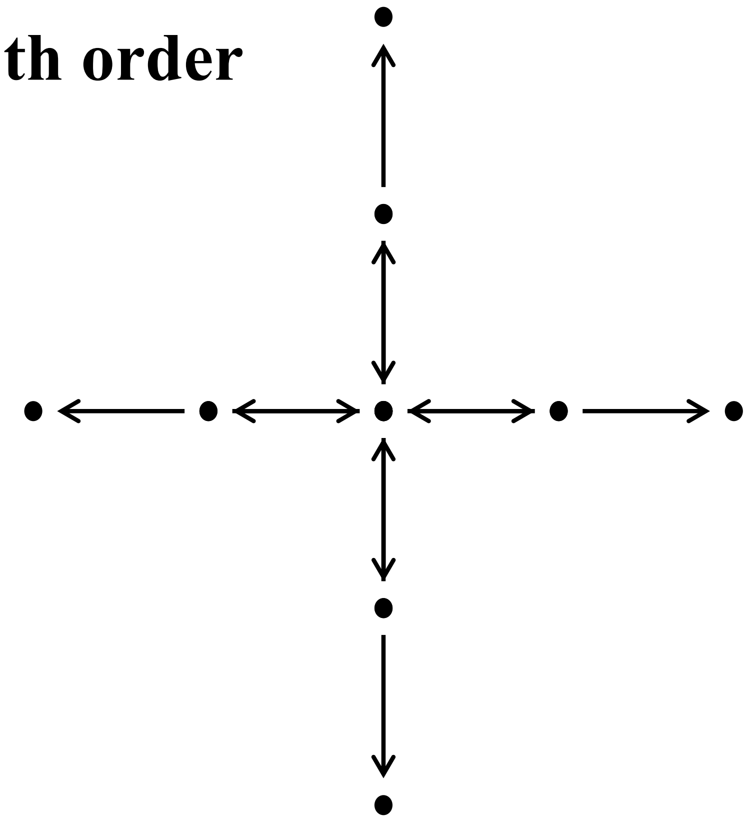
- Conserve energy and enstrophy in order to control nonlinear energy cascade; eliminate the need for numerical filtering to the extent possible.
- Conserve a number of first order and quadratic quantities (mass, momentum, energy, ...).
- Use consistent order of accuracy for advection and divergence operators and the omega-alpha term; consistent transformations between KE and PE in the hydrostatic limit.
- Preserve properties of differential operators.

NMM



Advection and divergence operators – each point talks to all eight neighboring points (isotropic)

Formal
4th order



NMM Vertical Coordinate

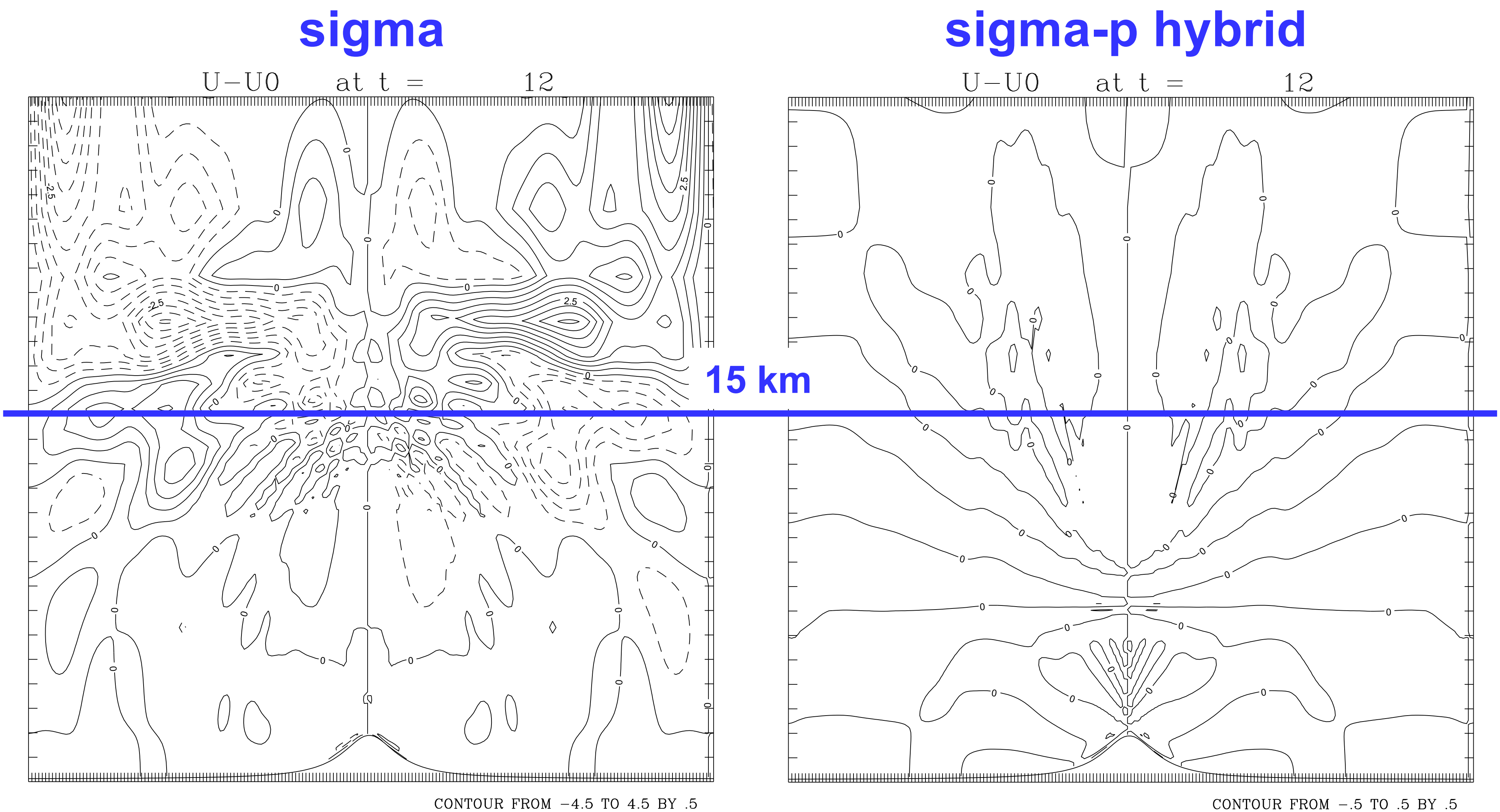
Pressure-sigma hybrid (Arakawa and Lamb, 1977)

Has the desirable properties of a terrain-following pressure coordinate:

- Exact mass (etc.) conservation
- Nondivergent flow remains on pressure surfaces
- No problems with weak static stability
- No discontinuities or internal boundary conditions

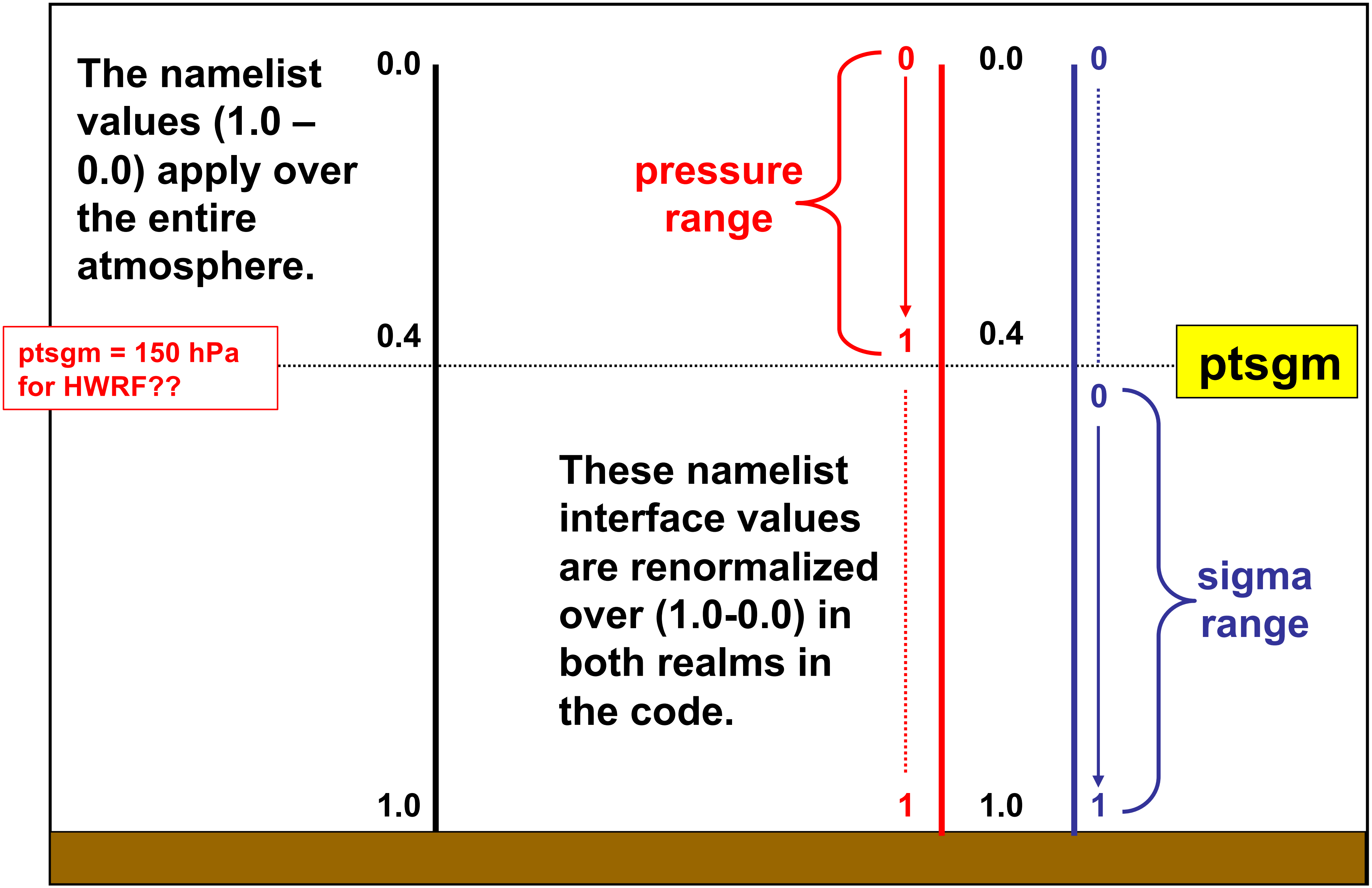
And an additional benefit from the hybrid:

- Flat coordinate surfaces at high altitudes where sigma problems worst (e.g., Simmons and Burridge, 1981)

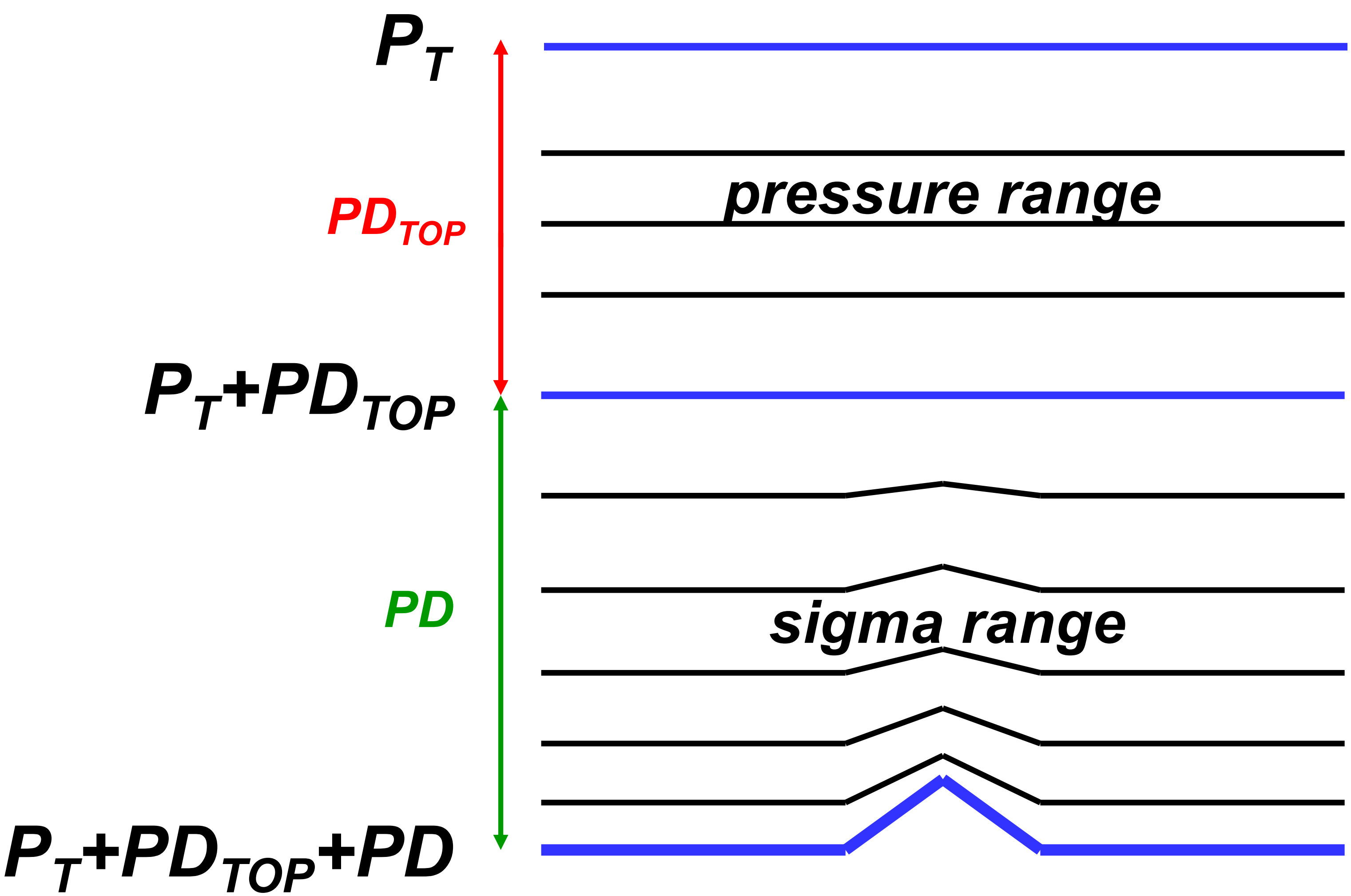


Wind developing due to the spurious pressure gradient force in an idealized integration. The hybrid coordinate boundary between the pressure and sigma domains is at ~400 hPa.

Pressure-Sigma Hybrid Vertical Coordinate



Pressure-Sigma Hybrid Vertical Coordinate



$$eta_2 = 0$$
$$0 < eta_1 < 1$$

$$eta_1 = 1$$
$$0 < eta_2 < 1$$

$$\pi = eta_1 * PD_{TOP} + eta_2 * PD + P_T$$

Lateral Boundary Conditions

- Lateral boundary information prescribed only on outermost row:

...	**H**	**v**	**H**	**v**	**H**	...	Pure boundary information
...	**v**	**H**	**v**	**H**	**v**	...	Avg of surrounding H points (blends boundary and interior)
...	**H**	**v**	**H**	**v**	**H**	...	Freely evolving
...	**v**	**H**	**v**	**H**	**v**	...	

- Upstream advection in three rows next to the boundary
  - No computational outflow boundary condition for advection

- Enhanced divergence damping close to the boundaries.

Dissipative Processes – lateral diffusion

A 2nd order, nonlinear Smagorinsky-type horizontal diffusion is utilized:

- Diffusion strength a function of the local TKE, deformation of the 3D flow, and a namelist-specified diffusion strength variable (*coac*).
- Lateral diffusion is zeroed for model surfaces sloping more than 4.5 m per km (0.0045) by default.
- *This slope limit can be adjusted with the namelist variable slophc. slophc is expressed as sqrt(2) times the true slope (making the 0.0045 default ~0.00636)*

Dissipative Processes - divergence damping *

- Internal mode damping (on each vertical layer)

$$\boxed{\mathrm{v}_j = \mathrm{v}_j + \frac{(\nabla \cdot dp_{j+1}\vec{\mathbf{v}}_{j+1} - \nabla \cdot dp_{j-1}\vec{\mathbf{v}}_{j-1})}{(dp_{j+1} + dp_{j-1})} \cdot DDMPV}$$

- External mode damping (vertically integrated)

$$\boxed{\mathrm{v}_j = \mathrm{v}_j + \frac{(\int \nabla \cdot dp_{j+1}\vec{\mathbf{v}}_{j+1} - \int \nabla \cdot dp_{j-1}\vec{\mathbf{v}}_{j-1})}{(\int dp_{j+1} + \int dp_{j-1})} \cdot DDMPV}$$

$$DDMPV \approx \sqrt{2} \cdot dt \cdot CODAMP$$

CODAMP is a namelist controlled variable = 6.4 by default.

A few NMM dynamics namelist switches

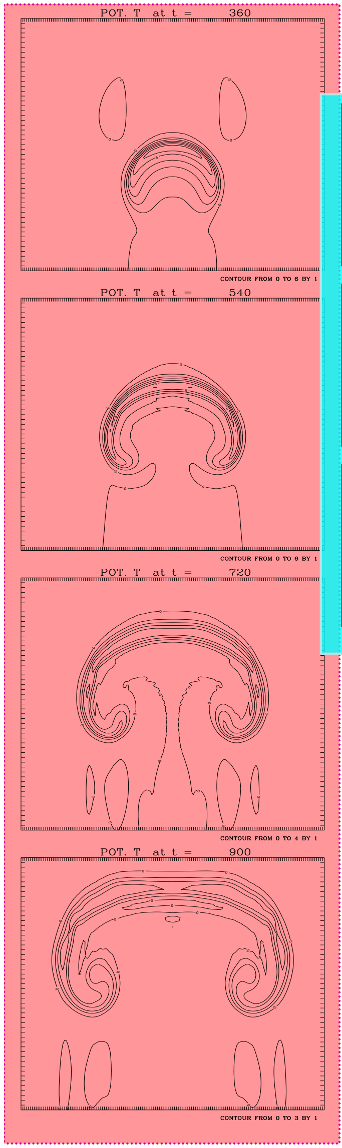
```
&dynamics
wp                                  = 0.00
coac                                = 0.75,
codamp                              = 6.4,
slophc                              = 0.006364
```

Defaults

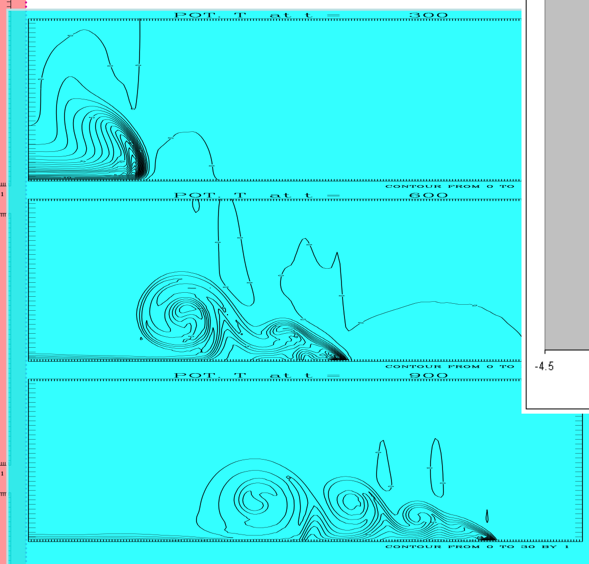
- **WP** - off-centering weight in nonhydrostatic computation (value of ~0.1 improves stability of some sub-1.5 km grid forecasts).
- **COAC** - diffusion strength (larger → more diffusive smoothing)
- **CODAMP** - divergence damping strength (larger → more damping, fewer small-scale regions of divergence).
- ***SLOPHC*** *- max surface slope for diffusion (larger value applies lateral diffusion over more mountainous terrain). [not specified in hwrf.conf file – Registry default used]*

Dynamics formulation tested on various scales

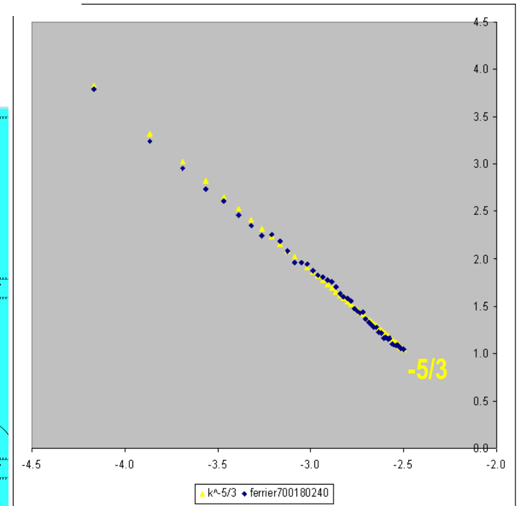
Warm bubble



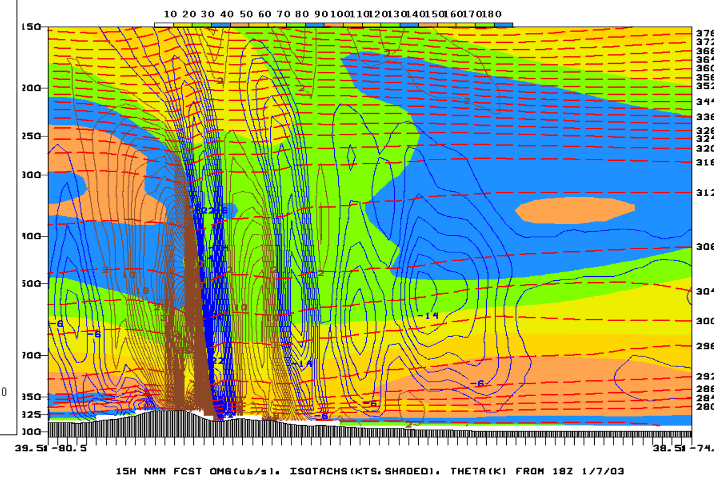
Cold bubble



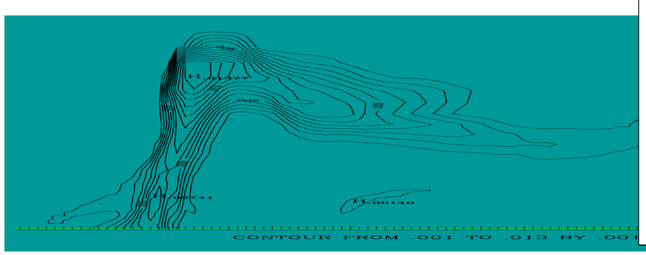
Decaying 3D turbulence



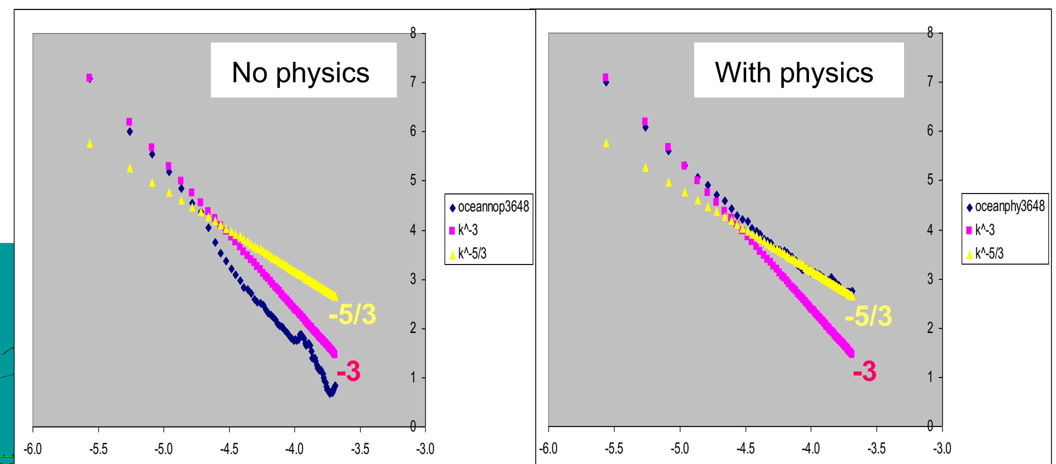
Mountain waves



Convection



Atmospheric spectra



Summary

- Robust, reliable, fast
- Represents an extension of NWP methods developed and refined over a decades-long period into the nonhydrostatic realm.
- Operational as the hurricane HWRF in 2007
- Utilized at NCEP in the HWRF, and is the basis for the NMMB model used by NAM, Hires Window and Short Range Ensemble Forecast (SREF) operational systems.

Backup slides